REMARKS

1. Drawings

Figures 1-4 and 7-8 have been amended. "Prior Art" is included in the legend. deleted sheets are marked "Deleted Sheet" in the top header, and replacement sheets are marked "Replacement Sheet" in the top header. Figures 4,6 have been amended to change the red chart lines with black.

2. Specification

On page 1 your corrections "a-posteriori" and (BEM)" have been made.

On page 2 line 1 has been deleted as recommended as well as additional non-relevant text. The reference to "k" has been deleted to avoid confusion and lines 7-11 which are lines 15-21 in the amended specification now reads "Maximum likelihood ML decisioning metrics DM are currently used in turbo and convolutional decoding wherein DM is the natural logarithm of the ML probability which is the conditional Gaussian probability of the observed communications channel output symbol y assuming that the transmitted symbol is x." and which definition of y,x should now agree with Hagenauer et. al. The DM definition now reads "$DM(y,x) = -|y-x|^2/2\sigma^2$ wherein the constant $1/2\sigma^2$ scales the DM relative to the noise floor $2\sigma^2$ of the symbol detector". This scaling constant in the DM metric ensures the metric is referenced to the noise floor of the receiver.

On page 5 the lines 9-12 which are lines 6-10 in the amended specification have been deleted to remove your objections. The index "k" is defined on page 4 in line 34 and on page 5 in lines 1-2 to be equal to the index of received codewords, trellis decoder states, and decoding clock. In lines 18-20 the y(k), y(j<k), y(j>k) are defined. Inclusion of the bit reference "b" for each codeword is deferred to later in the specification where it is used in the algorithms.

On page 6 your welcomed corrections for parentheses have been made. In applying Bayes rule to line 31 which is on page 8 in line 9 in the amended specification you correctly observed

$$p(s',s,\ y(j<k),y(k)) = p(s,y(k)|s',y(j<k))\ p(s',y(j<k))$$

and since the channel is "memoryless" the {s,y(k)} depend on the previous state s' and do not depend on the previous observed symbols $y(j<k)=\{y(1),...,y(k-1)\}$ which allows this equation to be reduced to

$$p(s',s,\ y(j<k),y(k)) = p(s,y(k)|s')p(s',y(j<k))$$

which is the equation in the specification.

On page 7 in line 7 which is on page 8 in line 16 in the amended specification the equation

$$\gamma_k(s,s') = p(s,y(k)|s')$$

remains valid since the "y(j<k)" has been deleted in the above equation on your "line 34" by applying the

"memoryless" channel assumption and it is gratifying to know that this equation agrees with Hagenaurer et. al.

On page 9 in linw 5 which is on page 10 in line 28 in the amended specification the equation

$$\gamma_k(s,s') = p(s,y(k)|s')$$

remains valid since the "y(j<k)" has been deleted in the above equation on your "line 34" by applying the "memoryless" channel assumption.

On page 10 which is page 13 in the amended specificationm your corrections have been made to "quadrature noise", "log, $\underline{\gamma}_k(s,s')$, of $\gamma_k(s,s')$", "log, $\underline{p}(d(k))$, of $p(d(k))$" and in lines 28 and 30 which are page 13 lines 29 and 31 the underlining remains since the "Log, $\underline{\alpha}_k(s)$, of $\alpha_k(s)$" and "Log, $\underline{\beta}_{k-1}(s')$, of $\beta_{k-1}(s')$" make it clear that the referenced equations require the underlining.

On page 16 which is page 19 in the amended specification your corrections have been made to the "a new" and the "framework" replaces my misspelled "paridyms".

On page 17 which is page 20 in the amended specification your correction "ratio of the" has been removed since it is incorrect.

On page 18 which is page 20 in the amended specification the "p(x|y)" and "p(y|x)" now replace

"f(x|y)" and "f(y|x)" and throughout the specification, The "the new MX" now replaces "our new MX' throughout the specification. In line 15 which is in line 15 in the amended specification "the maximum a-posteriori probability MX" now replaces "the maximum a-posterior probability MX". In line 16 the misspelled word "densify" has been eliminated. The amended "a-posteriori probability function p(x|y) of x conditioned on the observation y, with respect to the selection of x" is now clear with the replacement of "f(x|y)" with "p(x|y)". In lines 18-22 the reference statement has been amended to read "Maximizing MX is equivalent to maximizing the natural log, p(x|y), of p(x|y) which is the new decisioning metric $DX=Re(yx^*)/\sigma^2-|x|^2/2\sigma^2+\underline{p}(x)$ wherein $\underline{p}(x)$ is the natural log of the a-priori probability p(x), Re(o) is the real part of (o), and x* is the complex conjugate of x" which corrects my various mistakes including a verbose statement, un-matched left square bracket, use of "f" rather than "p", extraneous variable d=x, and the suggestion that there were multiple additive constants. In lines 30-32 the amended specification has deleted the "It will be proven that the MX is equivalent to ML and that maximizing DX is equivalent to maximizing DM for decisioning, with an added improvement in BER performance using DX" to avoid contradiction between the claimed "improvement in BER performance" and the stated equivalence when comparing the metrics..

On page 21 in lines 8:8 which is page 24 in lines 22-23 in the amended specification, the "DX=ln[p(x|y)]" includes the a-priori probability p(x) and there is no additive constant whereas "DM=ln[p(y|x)]" does not include p(x) and deletes the additive constant. Remaining text in question has been deleted. The "our" has been deleted throughout the specification.

On page 23 in lines 4-6 which is on page 27 in lines 11-13 in the amended specification the sentence in question "The equation DX=ln[] takes into account that the a-priori probability p(x)= factor is deleted and the additive constants are deleted" has been deleted since the "DX" now includes the probability factor p(x) and there is no additive constant in the calculation of "DX". The "p(o)" replaces "f(o)" throughout the specification to remove this source of confusion.

On page 24 in lines 16-17 which is on page 28 in lines 28-29 in the amended specification the "events {s',y(k)}, {y(j>k)} are independent since the channel is memoryless" is valid since state s' refers to k-1, y(k) refers to k whereas y(j>k) refers to k+1,k+2,. . . and only depends on the state transition $S_{j-1} \rightarrow S_j$ and cannot depend on $s'=S_{k-1}$ or y(k). A memoryless channel in coding theory means that the ML probability p(y|x) can be written as the factored Markov product

$$p(y(k)\,|\,S_k) = \prod_{i=1}^{k} p(\, y(i)\,|\,S_{i-1} \text{->} S_i\,)$$

wherein S_x=x(k) is the assumed state at k or equivalently the assumed value of x at k and also means that the y(k), k=1,2,...,N are independent. For example, with a constant data input of 1's or o's a correctly designed convolutional encoder will generate a random sequence for y(k), k=1,2,…,N.

On page 24 in lines 24-26 which is on page 29 in lines 5-7 the new a-posteriori recursive estimators $a_{k-1}(s')$, $b_k(s)$, $p_k(s|s')$ are related to he current maximum likelihood

ML estimators $\alpha_{k-1}(s')$, $\beta_k(s)$, $\gamma_k(s,s')$ by the following equations.

`1) The $\alpha_{k-1}(s')$ and $a_{k-1}(s')$ are related by:

$$\begin{aligned}\alpha_{k-1}(s') &= p(s',y(j<k)) \\ &= p(s'|y(j<k))\ p(y(j<k)) \text{ by Bayes theorem} \\ &= a_{k-1}(s')\ p(y(j<k))\end{aligned}$$

2) The $\beta_k(s)$and $b_k(s)$ are related by:

$$\begin{aligned}\beta_k(s) &= p(y(j>k)|s) \\ &= p(s|y(j>k)p(y(j>k))/p(s) \text{ by Bayes theorem} \\ &= p(s|y(j>k))p(y(j>k))/p(s) \\ &= b_k(s)\ p(y(j>k))/p(s)\end{aligned}$$

3) The $\gamma_k(s,s')$ and $b_k(s)$ are related by:

$$\begin{aligned}\gamma_k(s,s') &= p(s,y(k)|s') \\ &= p(y(k)|s',s)\ p(s|s') \text{ by Bayes theorem} \\ &= ML\ p(d(k)) \text{ since } p(d(k)\equiv p(s|s/) \\ &= p(s|s',y(k))p(s',y(k))/p(s') \text{ by Bayes theorem} \\ &= p_k(s|s')\ p(y(k)|s')) \\ &= p_k(s|s')\ p(y(k)) \text{ since } y(k) \text{ is independent}\end{aligned}$$

4) Combining equations (1),(2),(3) yields

$$\begin{aligned}p(s,s',y) &= \alpha_{k-1}(s')\beta_k(s)\gamma_k(s,s')p(d) \text{ using equations 4 in equations (3)} \\ &= a_{k-1}(s')\ b_k(s)\ p_k(s|s')\ p(y)/p(s) \text{ using equations (1),(2),(3) above} \\ &= p(s,s'|y)p(y)/p(s) \text{ using equations 4 in equations (12)}\end{aligned}$$

where p(s) is one of the two scaling factors between

the ML and MX since by definition

$$MX/p(s) = ML\ p(d(k))$$

and taking the natural log yields the equivalent relationship between the decisioning metrics Dx and DM

$$DX = DM + \ln[p(d(k))] + \ln[p(s)]$$

and which means the new turbo equations use DX throughout in place of DM and with some performance improvement.

What Haugenauer observes and which is reviewed in equations (6) and (9) is the well known observation that the MAP equation

$$L(d|y)=\ln[\ p(d=1,y)/p(d=-1,y)\ ]$$

in **3** in equation **(8)** reduces to the ML estimators α,β,γ in the numerator and demoninator

$$\begin{aligned} L(d|y)= &\ \ln[\ \Sigma_{(s,s'|d(k)=+1)}\ p(s,s',y)] \\ &-\ln[\ \Sigma_{(s,s'|d(k)=-1)}\ p(s,s',y)\ ] \end{aligned}$$

$$\begin{aligned} &=\ln[\Sigma_{(s,s'|d(k)=+1)} \exp(\underline{\alpha}_{k-1}(s')+DM(s|s')+\underline{p}(d(k))+\underline{\beta}_k(s))] \\ &-\ln[\Sigma_{(s,s'|d(k)=-1)} \exp(\underline{\alpha}_{k-1}(s')+DM(s|s')+\underline{p}(d(k))+\underline{\beta}_k(s))] \end{aligned}$$

whereas the new MAP equations in this specification in equations (18) is

$$L(d|y) = \ln[\ \Sigma_{(s,s'|d(k)=+1)}\ p(s,s'|y)] - \ln[\ \Sigma_{(s,s'|d(k)=-1)}\ p(s,s'|y)\ ]$$

$$= \ln[\Sigma_{(s,s'|d(k)=+1)} \exp(\underline{a}_{k-1}(s') + DX(s|s') + \underline{b}_k(s))] - \ln[\Sigma_{(s,s'|d(k)=-1)} \exp(\underline{a}_{k-1}(s') + DX(s|s') + \underline{b}_k(s))]$$

which is a distinctly different mathematical formulation when considering the differences in the estimator equations (1),(2),(3) above.

On page 32 in line 19 which is on page 38 in line 15 in the amended specification the correction has been made "ln[p(y(j>k-1),s')].

On page 35 in line 10 which is on page 41 in line 22 in the amended specification the obscure text has been changed to remove the elliptical objections, and the missing has been added where needed.

3. Claim Rejections - 35 USC § 112

The current amended claims 1-3 are believed to meet the requirements of 35 U.S.C. 112 "The specification shall conclude with one or more claims particularly pointing out and distinctly claiming the subject matter which the applicant regards as his invention".

4. Claim Rejections - 35 USC § 112

Claims 1-3 have been amended to particularly point out and distinctly claim the subject matter which I regard as the invention.

Recommended corrections have been made to the amended claims 1-3 including "and which comprises:" replacing "and which", "provide a means for" has been replaced by "using or said or evaluating", the expression "means for" has been deleted, "framework" replaces the incorrectly spelled word "paradym", and "x(k)" replaces "x(x)".

5. Claim Rejections - 35 USC § 103

It is clear that the subject matter of my patent is not "obvious at the time the patent was made to a person having ordinary skill in the art to which said subject matter pertains" from the discussion in the following regarding Dabiri"s patent for binary phase shift keying BPSK convolutional decoding.

6. Claim Rejections - 35 USC § 103

Dabiri patent no. 3,815,515 discloses the linearization of the ML decisioning metric DM in its reduced format which everyone uses $DM = -|x-y|^2/2\sigma^2$ for the implementation of decoding for a binary phase shift keying BPSK signaling. This linearization has been well known since the 70's and the reason it had never been used is that there is no savings in computational complexity.

Consider the DM calculation

$$\begin{aligned} DM &= -|x-y|^2/2\sigma^2 \\ &= -(x-y)(x-y)^*/2\sigma^2 \end{aligned}$$

$$= - \{ [Re(x-y)]^2 + [Im(x-y)]^2) / 2\sigma^2$$

wherein the number of real multiplies "M" or equivalently divides is DM(M)

DM(M) = 3M assuming that "$2\sigma^2$" is available

which means 3 real multiplies suffices. On the other hand consider the linear metric of Dabiri denoted by $\underline{DM}$

$$\underline{DM} = DM$$
$$= Re(y\ x^*)/\sigma^2 - |x|^2/2\sigma^2 - |y|^2/2\sigma^2$$
$$= Re(y\ x^*)/\sigma^2 - |x|^2/2\sigma^2 \text{ neglecting the additve term common to the decisioning metrics}$$
$$= [Re(y)Re(x)+Im(y)Im(x)] / \sigma^2 - |x|^2/2\sigma^2$$

wherein the number of real multiplies M or equivalently divides is $\underline{DM}$(M)

$\underline{DM}$(M) = 3M assuming that "$2\sigma^2$" is available

assuming that $|x|^2$ is available from a table lookup and the divide required to calculate $|x|^2/2\sigma^2$ has been made offline to the calculation of $\underline{DM}$. Note that $|x|^2/2\sigma^2$ for quadrature amplitude modulation QAM depends on the assumed symbol value in the trellis decoding. A similar calculation indicates that the number of real adds "A" or oequivalent subtracts are DM(A)=3A and $\underline{DM}$(A)=2A which is a relatively small difference. So there wqs no reason to consider $\underline{DM}$ for convolutional decoding and no motivation to consider $\underline{DM}$ for turbo and convolutional decoding.

A-posteriori statistic $p(x|y)$ had never been considered in turbo and convolutioonal decoding primarily because of the complexity in using $p(x|y)$ for communications and statistical estimation and because the ML statistic $p(y|x)$ is simple to use and is adequate. I knew that the $p(x|y)$ would provide some improvement in decoder performance but that prior efforts had discouraged its applicability due to the increased complexity for the relatively small performance improvement which some researchers found to be on the order of 0.25 dB in bit error rate BER improvement. However, with a reformulation of the turbo and convolutional decoding using a-posteriori statistics I was able to use the DX statistic without a computational penalty or an approximation and with a significant improvement on the order of 1.7 dB.

The development and application of the a-posteriori decisioning metric DX which results from using the a-posteriori statistic $p(x|y)$

$$DX = = \mathrm{Re}(y\ x^*)/\sigma^2 - |x|^2/2\sigma^2 + \ln[p(d)]$$

in turbo decoding and convolutional decoding in support of turbo decoding is a first and is certainly non-obvious from Dubiri's reduction of the ML decisioning metric $\underline{DM}$ to a linear form even though they are related

$$DX = \underline{DM} + \ln[p(d)]$$

7. Conclusions

The prior art of record which you found is included in the references "Cross-reference to related applications". The abstract has been amended and reduced in size to be within 150 words.

8. Conclusions

Thanks ever for your welcomed suggestions and guidelines.

Please give me any assistance you believe is necessary.

Sincerely, Urbain A. von der Embse

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